Minimal energy dissipation in classical and quantum physics

Latest news from Landauer's principle

Marco Pezzutto and Yasser Omar

Physics of Information Group, Instituto de Telecomunicações
and Instituto Superior Técnico, Universidade de Lisboa, Portugal

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Outline

- Classical Landauer's principle
  - Logical irreversibility
  - Physical irreversibility
  - Heat dissipation
- Basics of quantum mechanics
  - How to describe quantum states: state vectors
  - Superposition and symmetry principles
  - Measurement
  - Schrödinger equation and reversible evolution
  - Von Neumann entropy - Quantifying information
  - Bipartite systems - Quantifying correlations
- Landauer's principle in quantum physics
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Classical Landauer's principle

What are the ultimate physical limitations to reducing the dissipation in computing?

- Logical irreversibility in computing not only is *unavoidable*, but is also to some extent **necessary**
- Logical irreversibility $\Rightarrow$ Physical irreversibility
- Physical irreversibility $\Leftrightarrow$ Entropy increases

$\Rightarrow$ **Heat generation**

- Each **bit reset operation** is accompanied ed by a heat production of at least

$$\Delta Q = \kappa T \log 2 \,,$$

$\kappa =$ Boltzmann constant $\simeq 1,380 \times 10^{-23}\,\mathrm{J\,K^{-1}}$

Irreversibility

Logical irreversibility A logical/computing process is irreversible when **the output is not sufficient to reconstruct unambiguously the input.**

Example: *"And"* gate

IN		OUT
0	0	0
1	0	0
0	1	0
1	1	1

Physical irreversibility A physical process is irreversible if **its time-reversed process is forbidden by the II law of thermodynamics.**
Overall *increasing of the entropy*.

Example: Isothermal compression of a gas in a piston with *friction*

- It seems not possible to design a unique *conservative* process to reset a bit in its “0” or “1” state to, say, “1” state, *regardless of the initial state*
- Logical reversibility requires *storage of extra information at each step*
  $\Rightarrow$ Outgrowing resources needed
  $\Rightarrow$ It is impossible to execute *non-terminating programs*
- To load the program, all the needed bits have to be **reset**
  The *irreversible reset* operation is just moved at the beginning of the program!

- System + thermal reservoir at temperature T,
  $\delta Q =$ heat absorbed by the system from the reservoir,

$$\text{Entropy} \quad S_{A,B} := \int_{A,rev}^{B} \frac{\delta Q}{T}$$

- **First law**

$$dE = \delta Q - \delta L = TdS - \delta L$$

$\Rightarrow$ if energy is conserved we have the connection

$$\delta Q = TdS = \delta L$$

- **Second law**

$$dS_{\text{tot}} = dS_{\text{Sys}} + dS_R, \qquad dS_R = -\frac{\delta Q}{T}$$

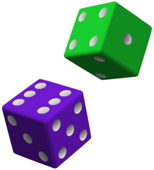
$$dS_{\text{tot}} \geq 0$$

Boltzmann entropy

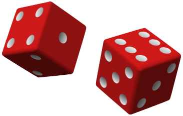
$$S = \kappa \log W$$

W = Number of microstates compatible with the given macrostate (*volume in the phase space*)

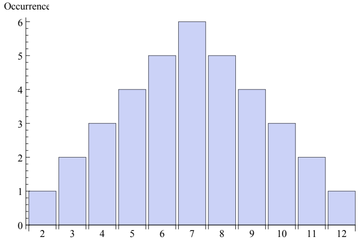
Example: two dices

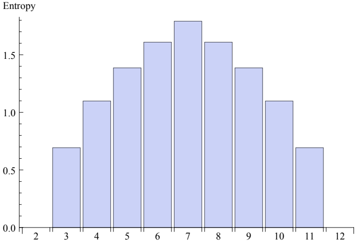


$$\begin{pmatrix} 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 5 & 6 & 7 & 8 & 9 \\ 5 & 6 & 7 & 8 & 9 & 10 \\ 6 & 7 & 8 & 9 & 10 & 11 \\ 7 & 8 & 9 & 10 & 11 & 12 \end{pmatrix}$$



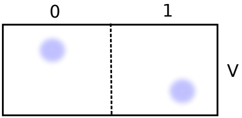
Microstate: numbers in each dice
Macrostate: the sum of them





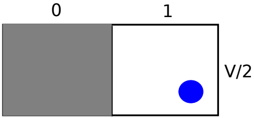
- The greater the entropy S, the greater the **ignorance** we have about the actual state of the system.
- **Example: a bit represented by a particle in a box**

Unknown bit value



$W_1 \propto V$

Bit reset to "1"



$W_2 \propto \frac{V}{2}$

$$\Delta S = S_2 - S_1 = \kappa \log \frac{W_2}{W_1} = \kappa \log \frac{V/2}{V} = -\kappa \log 2$$

- In order to have $dS_{\text{Sys}} + dS_R \geq 0$, the environment entropy should *increase of at least* $\kappa \log 2$
- At least a heat $\Delta Q = \kappa T \log 2$ must be dissipated

- Logical irreversibity $\Rightarrow$ Physical irreversibility
- Entropy generation $\Rightarrow$ Heat dissipation:

$$\Delta Q \geq \kappa T \log 2 \quad \text{per bit-reset operation}$$

The principle is based on the equivalence between thermodynamical and statistical entropy

- Orders of magnitude:
  - at room temperature, $\kappa T \log 2 \simeq 2.9 \times 10^{-21}$ J
  - 1 eV $\simeq$ 1.6×10^{-19} J
  - Current dissipation levels: $10^{-17} \sim 10^{-16}$ J,
    4 to 5 orders of magnitude above the Landauer's bound!

R. Landauer, *Irreversibility and Heat Generation in the Computing Process*, IBM J. Res. Develop. Vol. 5 No. 3, 1961

Basics of quantum mechanics

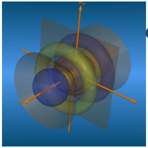
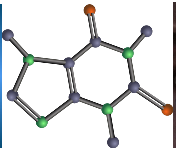
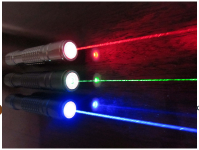
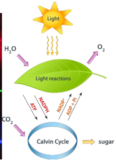
A word of caution:

"If quantum mechanics hasn't profoundly shocked you, you haven't understood it yet."

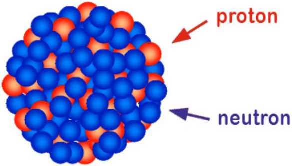
Niels Bohr

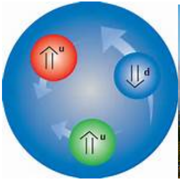
What is quantum mechanics?

The physical theory we employ to describe nature at the atomic scale and beneath

. . . electrons in atoms and molecules, chemical bonds, lasers and light-matter interaction



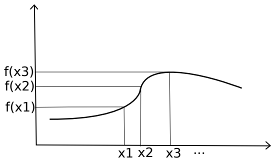



nuclear physics, elementary particle physics, nuclear energy . . .

- In classical mechanics - The state of a point-like particle: 6 real coordinates, the *position* $\vec{q}$ and the *momentum* $\vec{p}$. The space of the states is $\mathbb{R}^3 \oplus \mathbb{R}^3$
- In quantum mechanics, the state is specified by the

normalized **Wave function**, or **state vector** $|\psi\rangle$

- $|\psi\rangle$ can be
  - a finite-component vector: $|\psi\rangle = (\psi_1, \psi_2, \ldots, \psi_n)$
  - an infinite-component vector: $|\psi\rangle = (\psi_1, \psi_2, \ldots)$
  - a "continuous-component" vector ... i.e. a *function*:



- No matter the dimension, the space of states $|\psi\rangle$ has always the property of being a **Hilbert space** $\mathcal{H}$[1]
- The space is on **complex numbers** (ie vectors have complex components, not just real)
- Notation - **Scalar product**

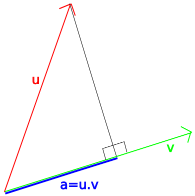
$$(\psi_1^*, \psi_2^*, \dots, \psi_n^*) \begin{pmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_n \end{pmatrix} = \langle\psi|\phi\rangle = a \in \mathbb{C}\,, \quad \langle\phi|\psi\rangle = a^*$$

- **Superposition principle** - If $|\psi\rangle$ and $|\phi\rangle$ are vector states, also

$$|\chi\rangle = |\psi\rangle + |\phi\rangle \quad \text{is a vector state}$$

[1] A *complete* abstract *vector space* with an *inner product* that allows *length and angle* to be measured.

- Projectors: operators $\mathcal{H} \to \mathcal{H}$, e.g. $\mathbb{C}^n \to \mathbb{C}^n$



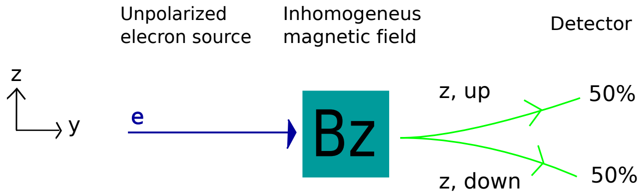
$$\hat{P}_v \left|u\right\rangle = a\left|v\right\rangle = (\langle v|u\rangle)\left|v\right\rangle = (\left|v\right\rangle\left\langle v\right|)\left|u\right\rangle$$

$$\Rightarrow \hat{P}_v = \left|v\right\rangle\left\langle v\right| \in \mathcal{M}_{n\times n}(\mathbb{C})$$

external product of v with itself:

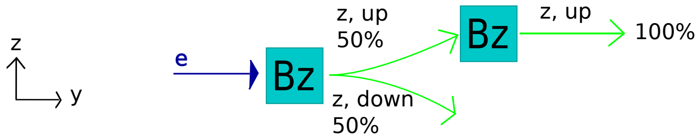
$$\left|v\right\rangle\left\langle v\right| = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} (v_1^*, v_2^*, \dots, v_n^*) = \begin{pmatrix} v_1 v_1^* & \dots & v_1 v_n^* \\ \vdots & & \vdots \\ v_n v_1^* & \dots & v_n v_n^* \end{pmatrix}$$

- **Spin: Intrinsic angular momentum** of a particle, even if *pointlike (!)* - Nothing rotates along any axis...
  $\Rightarrow$ Particles also have an **intrinsic magnetic moment**
- Magnetic moment - magnetic field interaction: $U_m = -\vec{\mu} \cdot \vec{B}$
  Force: $F_z = \partial(\vec{\mu} \cdot \vec{B})/\partial z = \mu_z \partial(B_z)/\partial z$
- Experimental setup and first measurement:



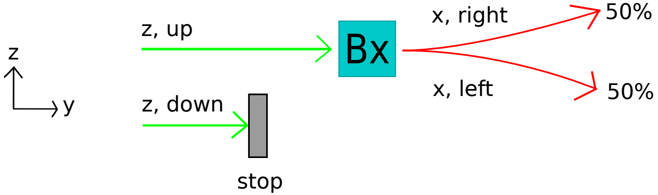
- First strangeness: only 2 outcomes! $\rightarrow$ *Space quantization*
- Let's denote the outcome states by $|{\uparrow_z}\rangle$ and $|{\downarrow_z}\rangle$

- Second measurement:



Seems obvious, but it turns there is something more . . .
The state changes to either $|\uparrow_z\rangle$ or $|\downarrow_z\rangle$!

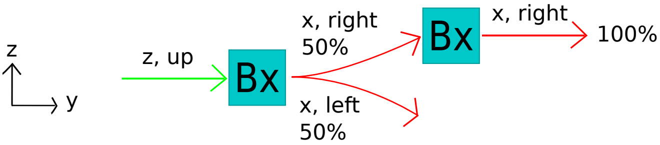
- Third measurement: rotation of the $\vec{B}$ field direction



Denote the x-outcome states by $|R_x\rangle$ and $|L_x\rangle$; it seems like

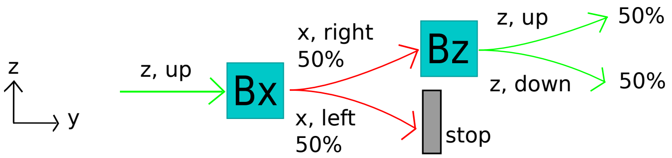
$$|\uparrow_z\rangle = (|R_x\rangle + |L_x\rangle)/\sqrt{2}$$

Of course, just like before,



$|\uparrow_z\rangle$ is a superposition of $|R_x\rangle + |L_x\rangle$, but
the measurement destroys the superposition
The state changes to either $|R_x\rangle$ or $|L_x\rangle$

- This is proven by the fourth measurement:



The $|\downarrow_z\rangle$ component has reappeared!
Now it seems like

$$|R_x\rangle = \frac{|\uparrow_z\rangle + |\downarrow_z\rangle}{\sqrt{2}}$$

- To summarize

$$\begin{cases} |R_x\rangle = \frac{|\uparrow_z\rangle + |\downarrow_z\rangle}{\sqrt{2}} \\ |L_x\rangle = \frac{|\uparrow_z\rangle - |\downarrow_z\rangle}{\sqrt{2}} \end{cases} \qquad \begin{cases} |\uparrow_z\rangle = \frac{|R_x\rangle + |L_x\rangle}{\sqrt{2}} \\ |\downarrow_z\rangle = \frac{|R_x\rangle - |L_x\rangle}{\sqrt{2}} \end{cases}$$

- We want to measure a quantity A that can assume a range of values $\{\alpha_j\}$ (finite, discrete or continuous)
  E.g. Energy E, momentum p, position $q \ldots$
- To the outcome set $\{\alpha_j\}$ we associate *a set of (orthogonal) vectors* $\{\,|a_j\rangle\,\}$
- If the system is prepared in one of these vector states, e.g. $|a_k\rangle$, a measurement of A gives the corresponding value α_k with probability 100 %
- Given the system prepared in a generic state $|\psi\rangle$, a single measurement of A will cause $|\psi\rangle$ to be projected on one of vectors $\{\,|a_j\rangle\,\}$, e.g.:

$$|\psi\rangle \rightarrow |a_1\rangle$$

$\Rightarrow$ the outcome of this measurement is α_1

Measurement changes the state

(Collapse of the wave function)

Repeating the process many times on identical copies of the initial state $|\psi\rangle$, we find on average each of the outcomes $\{\alpha_j\}$ appearing with probability $|\langle a_j|\psi\rangle|^2$

$$|\psi\rangle \rightarrow \begin{cases} |a_1\rangle & \text{with prob.} \quad |\langle a_1|\psi\rangle|^2 \\ \vdots & \\ |a_n\rangle & \text{with prob.} \quad |\langle a_n|\psi\rangle|^2 \end{cases}$$

- A first measurement of A gives, say, α_k;
  if we *repeat* the measurement, we find α_k with prob. 100 %
  Indeed the system state has changed to $|a_k\rangle$

- **Dynamics - Schrödinger equation**

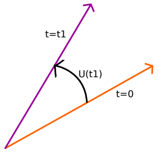
$$i\hbar \frac{\partial |\psi\rangle}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2 |\psi\rangle + V|\psi\rangle = \hat{H}|\psi\rangle$$

$\hat{H}$ = Hamiltonian, $\hbar$ = Plank constant $\simeq 1.054 \times 10^{-34}\mathrm{J}\cdot\mathrm{s}$

- Solution, isolated systems: **Reversible Unitary evolution**

$$|\psi_t\rangle = e^{-\frac{i}{\hbar}\hat{H}\,t}|\psi_0\rangle = \hat{U}_t|\psi_0\rangle$$

- *Unitary operators* **preserve norms and scalar products**



Complex equivalent of *rotations and changes of basis* for real vectors

- Time-reversed operator: $\hat{U}_t^\dagger = \hat{U}_t^{-1} = \hat{U}_{-t}, \quad \hat{U}_t\hat{U}_t^\dagger = \hat{U}_t^\dagger\hat{U}_t = \mathbb{I}$

- The collective wave function of a state of multiple identical particles must be either totally symmetric or totally antisymmetric under the exchange of any pair of particles
- Wave functions of many integer spin particles (bosons) are symmetric

$$\psi(1,2) \rightarrow \psi(2,1) = +\psi(1,2)$$

Wave functions of many half-integer spin particles (fermions) are antisymmetric

$$\psi(1,2) \rightarrow \psi(2,1) = -\psi(1,2)$$

- Pauli exclusion principle: since electrons are spin-1/2 particles, an atomic orbital can accommodate at most 2 electrons with opposite spin.
$\Rightarrow$ *one of the reasons why solid bodies cannot occupy the same place at the same time!*

- **Pure states:**

$$|\psi\rangle \leftrightarrow \hat{P}_\psi = |\psi\rangle\langle\psi|$$

- Ignorance about the exact system state: *statistical mixture*

$$\{p_j, |\psi_j\rangle\}, \quad p_j \in [0,1], \quad \sum_j p_j = 1$$

Mixed states:

$$\varrho = \sum_j p_j |\psi_j\rangle\langle\psi_j| \in \mathcal{M}_{n\times n}(\mathbb{C})$$

- Density matrix - Unitary evolution:

$$|\psi_{j,0}\rangle \to |\psi_{j,t}\rangle = \hat{U}_t\,|\psi_{j,0}\rangle$$

$$\varrho_0 \to \varrho_t = \hat{U}_t \varrho_0 \hat{U}_t^\dagger$$

- $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$, e.g. for the spins of two electrons, or the polarizations of two photons, $\mathcal{H}_A \otimes \mathcal{H}_B = \mathbb{C}^2 \otimes \mathbb{C}^2 = \mathbb{C}^4$
- Pure states: $|\psi_{AB}\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$, e.g.

$$|\psi_A\rangle = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \quad |\psi_B\rangle = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix},$$

$$|\psi_A\rangle \otimes |\psi_B\rangle = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \otimes \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} a_1 b_1 \\ a_1 b_2 \\ a_2 b_1 \\ a_2 b_2 \end{pmatrix}$$

- Mixed states:

$$\varrho_{AB} = \sum_{j,k} c_{j,k}\, \varrho_j^A \otimes \varrho_k^B$$

- Local states of ϱ_{AB} - Partial averages (partial trace)

$$\varrho_A = \text{average on } B \text{ of } \varrho_{AB} = \mathrm{Tr}_B[\varrho_{AB}]$$

$$\varrho_B = \text{average on } A \text{ of } \varrho_{AB} = \mathrm{Tr}_A[\varrho_{AB}]$$

Separable pure states: product of two local states

$$|\psi_{AB}\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$$

Entangled pure states: states that are **not separable**, e.g.

$$|\Phi^+\rangle = \frac{|\uparrow\rangle_A \otimes |\uparrow\rangle_B + |\downarrow\rangle_A \otimes |\downarrow\rangle_B}{\sqrt{2}}$$

Local states

- Separable states:

$$\begin{cases} |\psi_{AB}\rangle \rightarrow \text{average on A} \rightarrow |\psi_B\rangle & \text{pure} \\ |\psi_{AB}\rangle \rightarrow \text{average on B} \rightarrow |\psi_A\rangle & \text{pure} \end{cases}$$

- Entangled states:

$$|\Phi^+\rangle\langle\Phi^+| \rightarrow \text{average on B} \rightarrow \frac{|\uparrow\rangle_A\langle\uparrow|_A + |\downarrow\rangle_A\langle\downarrow|_A}{2} \quad \text{MIXED!}$$

What does this mean?
Given the AB state $|\Phi^+\rangle$, A measures on his side with B_z:

$$\begin{cases} \text{50\% } A \text{ finds } |\uparrow\rangle \Rightarrow \text{ collapse: } |\Phi^+\rangle \rightarrow |\uparrow\rangle_A \otimes |\uparrow\rangle_B \\ \text{50\% } A \text{ finds } |\downarrow\rangle \Rightarrow \text{ collapse: } |\Phi^+\rangle \rightarrow |\downarrow\rangle_A \otimes |\downarrow\rangle_B \end{cases}$$

Consequences:

- A and B each have **maximal ignorance** about the system state, even though the global state is pure
- A measure of A affects instantaneously what B will measure

Entanglement $\leftrightarrow$ Non local correlations

Given a mixed state $\varrho = \sum_j p_j |\psi_j\rangle\langle\psi_j|$, the Von Neumann entropy quantifies the **ignorance** we have about its state, the **randomness** of the statistical mixture it represents

Von Neumann Entropy

$$S(\varrho) = -\mathrm{Tr}[\varrho \log \varrho] \geq 0$$

If $\langle\psi_j|\psi_k\rangle = \delta_{jk}$ ($\{p_j\}$ are eigenvalues of ϱ), then $S(\varrho) = -\sum_j p_j \log p_j$

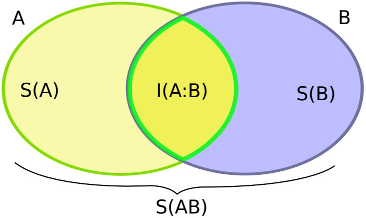
Some important properties:

- **Certainty**: if $\varrho = |\psi\rangle\langle\psi|$ (no statistical uncertainty) then $S(|\psi\rangle\langle\psi|) = 0$ and v/v
- **Additivity**: for a bipartite uncorrelated state $\varrho_{AB} = \varrho_A \otimes \varrho_B$,
$$S(\varrho_{AB}) = S(\varrho_A) + S(\varrho_B)$$
- **Maximum**: $S(\varrho)$ is maximum when the statistical mixture is the maximally random random one,
$$\text{all } p_j = \frac{1}{n} \quad \Rightarrow \quad -\sum_j p_j \log p_j = \log n$$

- Given a system A described by ϱ_A, if we perform a measurement it collapses on a vector $|\psi_A\rangle$, therefore now we know it **exactly.**
- $S(\varrho)$ quantifies the amount of **information we gain**:

$$S(\varrho_A) - S(|\psi_A\rangle\langle\psi_A|) = S(\varrho_A)$$

- Quantifying the **degree of correlation between two systems**: how much information can I learn about B, if I measure A?



MUTUAL INFORMATION

$$I(A:B) = S(\varrho_A) + S(\varrho_B) - S(\varrho_{AB}) = I(B:A) \geq 0,$$

$\varrho_{A,B} = \mathrm{Tr}_{B,A}[\varrho_{AB}]$, local states

Relative entropy

$$D(\varrho\|\sigma) = \mathrm{Tr}[\varrho \log \varrho - \varrho \log \sigma]$$

- $D(\varrho\|\sigma) \geq 0$ for all ϱ, σ
- $D(\varrho\|\sigma) = 0 \Leftrightarrow \varrho = \sigma$

Basics of quantum mechanics - Summary

- Physical states are represented by vectors in a Hilbert space
- Measurement changes the state (Wave function collapse)
- Superposition principle
- Measurement destroys superposition
- Schrödinger dynamics and unitary reversible evolution
- Mixed states: density matrix
- Bipartite states and local states
- Entanglement
- Quantifying information: Von Neumann entropy
- Quantifying correlations: mutual information

Landauer's principle in quantum physics

Does Landauer's principle apply
also at microscopic scales?

Recent proof and generalization of the Landauer's bound $\Delta Q \geq T \Delta S$ in a statistical physics framework

Assumptions:

- Process involves only system S + reservoir R, now both of finite dimension d_S and d_R (and nothing else!)
- System and reservoir are initially uncorrelated:

$$\varrho_{SR} = \varrho_S \otimes \varrho_R$$

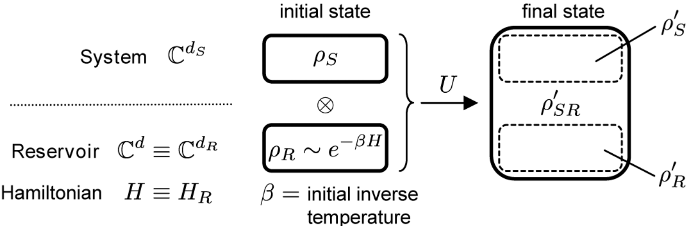
- The reservoir is initially in thermal equilibrium at temperature T:

$$\varrho_R = \frac{e^{-\beta H_R}}{\mathrm{Tr} e^{-\beta H_R}}$$

$\beta = 1/\kappa T$, inverse temperature,
$H_R =$ reservoir Hamiltonian

- Global unitary evolution:

$$\varrho'_{SR} = \hat{U}\varrho_{SR}\hat{U}^{\dagger}$$



Local final states: $\varrho'_{S,R} = \mathrm{Tr}_{R,S}[\varrho'_{SR}]$
S and R can develop classical or quantum correlations (entanglement)

Quantities involved:

- Entropy **decrease** of the system:

$$\Delta S = S_{in} - S_{fin} = S(\varrho_S) - S(\varrho_{S'}),$$

$S(\varrho) = -\mathrm{Tr}[\varrho \log \varrho]$ - Statistical/info-theoretical entropy

- **Heat** transferred **to** the reservoir - Thermodynamics

$$\Delta Q = E'_R - E_R = \mathrm{Tr}\big(H_R(\varrho'_R - \varrho_R)\big)$$

Quantum enunciation of Landauer's principle

$$\beta \Delta Q = \Delta S + I(S' : R') + D(\varrho'_R \| \varrho_R)$$

Since $I(S' : R') \geq 0,\ D(\varrho'_R \| \varrho_R) \geq 0,$

$$\beta \Delta Q \geq \Delta S$$

With few reasonable assumptions, the result is generalized to the case of an **infinite-dimensional reservoir**, closer to what one expects for a thermal bath:

- S and R both describable by separable Hilbert spaces
- $S(\varrho_S) < \infty$
- H_R bounded below $\Rightarrow S(\varrho_R) < \infty$

Some remarks:

- In this formulation, Landauer's principle is derived as a *consequence of the second law of thermodynamics*, formulated as

$$\big(S(\varrho'_S) - S(\varrho_S)\big) + \big(S(\varrho'_R) - S(\varrho_R)\big) \geq 0$$

- The connection between statistical/information theoretical entropy and thermodynamics through interpretation of $E_{R'} - E_R$ as **heat**

Reference: D. Reeb & M. M. Wolf, *(Im-)Proving Landauer's Principle*, arXiv:1306.4352v2

- System and reservoir initially uncorrelated
- Reservoir initially in a thermal state
- Global unitary evolution

Equality form of Landauer's principle

$$\beta \Delta Q = \Delta S + I(S' : R') + D(\varrho'_R \| \varrho_R)$$

$$\Rightarrow \quad \beta \Delta Q \geq \Delta S$$

It can be violated if **any** of the assumptions is dropped!

- Classical Landauer's Principle:
  logical irreversibility implies physical irreversibility and heat dissipation of $\kappa T \log 2$ per bit-reset operation
- Quantum mechanics:
  - measurement changes the state and destroys superposition
  - Schrödinger equation and unitary evolution
  - Quantifying information and correlations:
    Von Neumann entropy and mutual information
- Landauer's principle in quantum physics:
  a recent proposal in a statistical physics framework

- The classical version of Landauer's principle is based on the assumption of equivalence between thermodynamic and information theoretical entropy
  Is this assumption legitimate and reasonable?
- Given the energy scale indicated by the Landauer's bound ($\simeq 10^{-21}$ J) it is likely we cannot do without a quantum formulation
- **Landauer's bound as a goal** - Seems we still have a huge margin of improvement in current technologies, that dissipate much more than Landauer's bound
- **Landauer's bound as a challenge** - *On the theoretical side*: revisit the assumptions at the base of its formulation and possibly find other bounds
  On the experimental side: designing and implementing technologies and devices that approach (or even break) Landauer's bound

Landauer project

Thank you for your attention

yasser.omar@tecnico.ulisboa.pt
marco.pezzutto@tecnico.ulisboa.pt

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TÉCNICO LISBOA